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This report briefly summarizes the work performed for the Army Research Office under grant number ARO-DAAH04-93-G-0012 on the design and analysis of robust feedback systems. This research is broadly concerned with robust control and identification theory and applications. We have obtained significant new theoretical results and efficient computational algorithms for: $\mathcal{H}_2$, $\mathcal{H}_\infty$, and $\mathcal{H}_2/\mathcal{H}_\infty$ control synthesis, multiobjective robust control, robust identification and gain-scheduled control. The proposed methods are demonstrated in several engineering applications.

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CONTROL AND INDENTIFICATION OF UNCERTAIN SYSTEMS

FINAL PROGRESS REPORT

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MARCH 26, 1998

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DEPT. OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE
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Abstract

This report briefly summarizes the work performed for the Army Research Office under grant number ARO-DAAH04-93-G-0012 on the design and analysis of robust feedback systems. This research is broadly concerned with robust control and identification theory and applications. We have obtained significant new theoretical results and efficient computational algorithms for: $\mathcal{H}_2$, $\mathcal{H}_\infty$, and $\mathcal{H}_2/\mathcal{H}_\infty$ control synthesis, multiobjective robust control, robust identification and gain-scheduled control. The proposed methods are demonstrated in several engineering applications.

Contents

1 Introduction

Our research supported under this grant has focused on various problems in robust control and robust identification. We have worked on the following topics:

- Multiple objective robust controller synthesis
- Robust control analysis and synthesis
- Robust identification and control-oriented modeling
- Implementation of gain scheduled controllers for nonlinear control
- Engineering applications

Since much of our work has already appeared in a number of journal and conference publications, and in order to keep this report concise, we will simply summarize the main contributions in these various research directions. The interested reader can find complete details, precise mathematical formulations, and proofs in the papers cited in the report. Furthermore, again motivated by concerns for brevity, we will restrict the discussion entirely to our papers supported under this grant. For a complete discussion of relevant work by other researchers, we refer the reader to our publications cited below.

2 Multiple Objective Robust Controller Synthesis

Consider the feedback system shown in Figure 1. The plant to be controlled is denoted by G, while the controller is denoted by C. The exogenous inputs are $w_0, \ldots, w_s$ (these are signals such as sensor noises, load disturbances, commands, input channels for modeling uncertainty,) The controlled or regulated outputs are $z_0, \ldots, z_s$ (these signals represent weighted tracking errors, weighted actuator inputs, output channels for modeling uncertainty). The control input vector is u while the measured output vector is y. The closed-loop input-output operator from w_i to z_i will be denoted by $T_i(C)$.

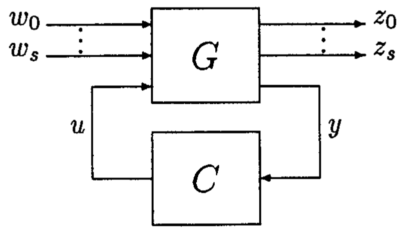

Figure 1: The synthesis framework

We are primarily interested in performance measures of the form

$$J_i(C) := \|T_i(C)\|_{\alpha_i},$$

where α_i indicates the norm of interest. Typically, $\alpha_i = 1, 2, \infty$. These norms are the most commonly used system norms in robust control.

Most of our work in this area has been focused on the so called mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problems. The basic notion is to minimize the $\mathcal{H}_2$ norm of a given closed loop transfer function subject to a constraint on the $\mathcal{H}_\infty$ norm of another closed loop transfer function. The $\mathcal{H}_\infty$ norm constraint may represent, for example, a robust stability constraint while the $\mathcal{H}_2$ norm objective function may represent a performance metric. Thus, a typical problem is to minimize J_1 subject to $J_2 < 1$ with $\alpha_1 = 2$ and $\alpha_2 = \infty$.

In previous work, we had given convex optimization based approaches to a version of this mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem in which the actual $\mathcal{H}_2$ norm is replaced by an upper bound on the $\mathcal{H}_2$ norm. The upper bound depends on the $\mathcal{H}_\infty$ norm constraint. The resulting problem then becomes a tractable approximation to the "exact" mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem. This approximation by initially formulated by D. Bernstein and W. M. Haddad and has been adopted by a number of researchers. In [18], we showed via some numerical examples, that the optimal controller for the approximate mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem can yield a $\mathcal{H}_2$ performance which is actually worse than the central $\mathcal{H}_\infty$ controller for the pure $\mathcal{H}_\infty$ suboptimal controller design problem. This is quite surprising in that the $\mathcal{H}_\infty$ central controller is completely independent of the $\mathcal{H}_2$ norm objective!! What this shows is that in optimizing an upper bound on the actual objective function one may do worse than

even ignoring it. The reason for this interesting behavior is that the upper bound approximation becomes quite bad as one gets close to the $\mathcal{H}_\infty$ norm constraint, as J_2 get s close to 1.

Most of the results in the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control theory are restricted to the case of one (vector) input and two (vector) outputs (or the dual problem of two vector inputs and one vector outputs.) In earlier work, we had obtained some sufficient conditions for the general two vector inputs and two vector outputs case. These results were further generalized in [10, 27]. This generalization combined the best features of our earlier results is perhaps the most general result of its kind in the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ theory.

In a tracking problem context, we posed the problem of robustly tracking sinusoidal reference signals in the presence of norm bounded unstructured uncertainty in [3]. A complete analytical solution was also given in this paper. The solution involves solving certain linear matrix equations and algebraic Riccati equations.

In [8], we gave a $\mathcal{H}_2/\mathcal{H}_\infty$ formulation of a filtering and estimation problem, and obtained results on the structure of the solution. We also provided a general convex optimization based solution to this problem. This paper also contains some promising simulation results showing robustness of the resulting filters to variations in the noise spectral properties.

3 Robust Control Analysis and Synthesis

This research project was concerned with extending recently developed robust control methods to a variety of new and technically challenging problems. These problems included sampled-data control, gap metric performance, real parametric uncertainty and decentralized control problems. In addition to developing computationally attractive and effective methods for these problems, our analysis has revealed an interesting theoretical properties concerning robustness and optimization.

Our research in robust sampled-data control was motivated by the widespread use of digital controller implementations. In the last few years, a number of researchers have focussed their efforts

on developing new techniques for the analysis and synthesis of robust sampled-data systems. This research direction has the promise of providing tools for the analysis and design of digital control systems and understanding tradeoffs between sampling rate, performance, and robustness.

Since frequency response occupies such a central place in the theory of control systems, we have developed procedures for computing frequency response of sampled-data systems [33, 15]. This is not straightforward because sampled- data systems are not time-invariant. But their periodic time-varying nature makes this feasible. In order to facilitate numerical computation of the frequency response, some upper bounds on the gain of the frequency response operator were obtained in [22]. These frequency response techniques are also very useful for signal reconstruction problems which arise in signal processing as shown in [26, 34]. This may develop into a nice new research direction.

A time domain model involving differential equations with discontinuous jumps for sampled-data systems was proposed in [14]. Based on this model, a complete characterization of the $\mathcal{L}_2$ norm of a closed loop sampled-data system was obtained in terms of Riccati differential and algebraic equations. These equations lead to state-space type computational procedures for the $\mathcal{H}_\infty$ norm in the sampled- data systems case.

A second research direction we pursued was in the stability and performance robustness analysis problems for real parametric uncertainty. Although it is well-known that real parametric uncertainty analysis is computationally difficult, we made some progress on this problem. First, we gave an analytical procedure to compute the average case and worst-case $\mathcal{H}_2$ norm of a closed loop system in the presence of real parametric uncertainty [4]. The results are particularly useful for one real parameter entering linearly in the systems matrices. This paper also introduced the interesting concept of average case analysis in a robust control context.

Nevertheless, it has been shown many researchers that the robust stability analysis against real perturbations problem is NP hard. However, in what may turn out to be a breakthrough direction, we took a probabilistic approach to this problem in [25]. We showed that there exist randomized algorithms that have only polynomial complexity. This work may start a new trend towards a more "experimental" approach to robust control theory.

A close relation between state-feedback controllers which optimize stability robustness in a gap metric setting and the classical LQR solution was given in [9]. This is of interest since it showed an additional interesting property of the LQR state-feedback controllers.

One approach to deal with transient response specifications is to impose a constraint on the location of closed loop poles. A problem of optimal $\mathcal{H}_2$ controller design subject to constraints on the closed loop pole locations was solved in [28, 13]. The solution involves solving a pair of algebraic Riccati equations, much like the classical LQG solution.

In a completely different direction, we [6] considered the problem of decentralized control and showed that under a very mild connectivity assumption, one can always stabilize a linear time-invariant plant using decentralized control using periodic feedback showing yet again the power of periodic feedback.

4 Robust Identification

In the previous period of this grant, we ban a study on identification problems in $\mathcal{H}_\infty$. The problem there was to obtain a model for the system from noisy frequency response data. This model was to include bounds on the modeling error which could be utilized for robust controller design and analysis. These error bounds depended both on the a priori information on the unknown system and data noise as well as the algorithm. We obtained some very effective and easily computable solutions to the problem of identification in $\mathcal{H}_\infty$. These algorithms have the merit of being computationally very efficient. They involve using the fast Fourier transform (FFT) algorithm and Hankel singular value computations. We also obtained error bounds on the modeling error.

In this phase of the grant, we extended the earlier $\mathcal{H}_\infty$ identification algorithms to consider the problem of nonuniformly spaced frequency response data [1]. This problem naturally arises both in continuous-time and discrete-time systems. An identification algorithm was presented in [1] along with a careful analysis of error bounds and convergence rates.

However, despite these results and considerable other work, these model error bounds do not appear to be useful from a practical point of view. Worst-case model error bounds are in general too conservative and require a priori information that is difficult to obtain. We have thus begun to explore alternative methods for estimating model uncertainty. One promising avenue for this is to combine system identification with model (in)validation. The idea is to postulate a model for the system along with bounds on the uncertainty. This could come from performing system identification experiments. Then with fresh data, one could pose the question whether the data is consistent with the postulated model. This precise scenario was treated in our prize winning paper [11] where we gave an analytical solution to a time-domain model validation problem.

One of major efforts under the current grant was to develop methodologies for making the algorithms for identification in $\mathcal{H}_\infty$ applicable to real data and systems. This was the topic of Ph D dissertation of Dr. Jonathan Friedman. Much of this work has been summarized in the applications papers [23, 5]. The focus of these papers is on the development of techniques to facilitate the application of algorithms for identification in $\mathcal{H}_\infty$ to real experimental data which can be collected using commercially available equipment such as frequency analyzers. In addition to these applications, R. Rajamani at GE has also applied these algorithms, with excellent results, to modeling a high power combustor.

In a more traditional direction, we analyzed the performance of the classical least squares algorithm in the presence of worst case bounded noise [2, 17]. After deriving some bounds on the parameter estimation error, we showed that the least squares algorithm is robustly convergent in the sense defined in the worst-case identification literature.

The last decades have seen tremendous progress in the design methodologies for linear and nonlinear control systems. Obviously, to utilize these advanced methods, it is necessary to have suitable models for the processes to be controlled. By comparison to controller synthesis, the progress in system identification and modeling has not been as vigorous. Therefore, we feel that it is important and useful, from an applications standpoint, to focus on the process of obtaining control oriented models for nonlinear systems. Thus, a major new direction in our research in system identification is focused on nonlinear systems. This has arisen out of our work in the

area of control of semiconductor manufacturing. Since first principles models are not available, we have been forced to use system identification techniques for this class of problems. In [32], we have developed practical techniques for the identification of nonlinear systems using Hammerstein models. The resulting models are in a form suitable for controller design.

5 Gain Scheduled Controllers for Nonlinear Control

Recently, we have obtained a method [7, 24] for implementation of gain scheduled controllers for nonlinear systems that has the useful property that the robustness and stability properties of the linear design are locally preserved for the closed loop nonlinear system. This technique is very easy to implement on real engineering systems. We have already used it a number of applications studies.

6 Engineering Applications

Several applications of the above control methods have been investigated. As mentioned above, the robust identification algorithms were tested on data from two experimental structures:

- Data from the ATB-100 test structure at the ARDEC, Picantinny Arsenal, NJ
- Data from a flexible structure at JPL, Pasadena, CA.

The results of these tests are discussed in the papers [21, 5, 23]. We have shown that the proposed algorithms produce very good models with efficient and standard computational tools.

A second major application effort has been in applying modern control methods is in the area of control of semiconductor manufacturing processes. Here we have been exploring a variety of issues including nonlinear system identification from real data, nonlinear control, [32] etc. An attractive feature of this activity is that we are able to implement and test or algorithms. This is enormously

important in that the insight offered by practical implementation can not be replaced by software experiments.

In addition, we are beginning to develop applications projects in the area of automotive systems control [12]. Given the proximity of the University of Michigan to the automotive industry, and the fact that control is becoming a major part of modern automobiles, we have been developing the infrastructure to enable automotive controls research. For this, we have been developing full-scale detailed nonlinear simulations of key automotive subsystems. In [12], we presented novel control strategy for reducing the slow variations in engine idle speed. We have also been exploring the use of randomized global optimization methods discussed in [25] for an optimal transmission control problem [35, 36].

7 List of Personnel Supported Under ARO Contract

1. Dr. P. P. Khargonekar
2. Dr. S. Rangan
3. Dr. J. Friedman

   Thesis Title: Modeling, Identification, and Control of Flexible Systems
4. Dr. A. Yoon

   Thesis Title: Randomized Algorithms and Global Optimization for Optimal Robust Control.

   Graduation Date: December 1997

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